Effect of Swirl on Jet Atomization

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Introduction

BECAUSE of its practical significance, the subject of jet atomization has been given considerable attention in the literature.[1,2] However, the number of published investigations on the problem of the effect of swirl on the atomization of a liquid jet has been very limited. The most comprehensive theoretical treatment of the problem is that of Ponstein.[3] More recently, Kang and Lin[4] studied the spatial instability of a swirling liquid jet, including the effect of nonaxisymmetric disturbances in their analysis. Lian and Lin[5] investigated the convective instability of a viscous liquid jet emanating into a swirling inviscid gas.

All of the previous studies of the effect of swirl on liquid jet atomization were conducted via the linear theory of hydrodynamic stability. The problem with the linear stability analysis is the assumption that the perturbations are infinitesimally small, since, after a finite interval of time, unstable perturbations will have grown to finite quantities.

Because of the limitations of the linear stability theory, alternative methods of analysis have been sought. These include the method of strained coordinates.[6-10] Direct numerical solution of Navier-Stokes equations in their axisymmetric form was attempted by Shokoohi and Elrod.[11] Bogy[1] used the Cosserat theory developed by Green.[12] A weakly nonlinear instability analysis was advanced by Ibrahim and Lin.[13] Though mathematically or computationally elegant, all of the aforementioned methods suffer inherent complexity.

Lee[14] developed a one-dimensional nonlinear direct-simulation technique that proved to be a simple and practical approach to investigate the nonlinear instability of a liquid jet. Lee's[14] direct-simulation approach formed the basis of a comprehensive treatment of the jet instability and atomization presented by Chuech et al.[15] In the present work we shall extend the direct-simulation analysis of Chuech et al.[15] to include the effect of swirl on jet atomization.

Analysis

The basic formulation of Lee[14] was adopted by Chuech et al.[15] in their analysis of the instability of an infinitely long cylindrical liquid jet. The liquid is assumed to be incompressible and inviscid. The axial velocity and the pressure are assumed to be constant over the cross section of the jet and dependent only on axial coordinate z and time t. These assumptions are consistent with the study of the case of long waves.[16] Only axisymmetric disturbances are considered in the analysis. In a cylindrical coordinate system moving at the unperturbed (basic) axial jet velocity U_0, a set of one-dimensional equations is derived:

Continuity:

$$\frac{\partial h^2}{\partial t} + \frac{\partial (h^2 u)}{\partial z} = 0 \tag{1}$$

Axial momentum:

$$\frac{\partial (h^2 u)}{\partial t} + \frac{\partial (h^2 u^2)}{\partial z} = -\frac{h^2}{\rho}\frac{\partial p}{\partial z} \tag{2}$$

with

$$p = p_g + p_\sigma \tag{3}$$

where h is the distance from the jet axis to its perturbed surface, u is the axial velocity, ρ is the liquid density, p is the perturbation pressure in the liquid, p_g is the perturbation pressure in the gas, and p_σ is the perturbation pressure due to surface tension:

$$p_\sigma = \frac{\sigma}{[1 + (\partial h/\partial z)^2]^{1/2}} \left\{\frac{1}{h} - \frac{(\partial^2 h/\partial z^2)}{[1 + (\partial h/\partial z)^2]}\right\} \tag{4}$$

where σ is the surface tension.

The initial conditions are given by

$$h(t = 0, z) = \eta_0 \cos(kz) + a, \qquad u(t = 0, z) = 0$$

where a is the unperturbed jet radius, η_0 is the amplitude of the initial perturbation, and k is the wave number. The computation domain is taken equal to one wavelength $\lambda = 2\pi/k$. Periodic boundary conditions are applied at the right and left ends of this domain. The previous equations are solved numerically, in their dimensionless form, by the upwind total variation diminishing (TVD) scheme with characteristic decomposition. All lengths are made dimensionless by the jet radius, time is made dimensionless by multiplying by the capillary velocity $(\sigma/\rho a)^{1/2}$ and dividing by the jet radius, and the pressure is nondimensionalized by the product of the liquid density and the square of the capillary velocity. Details of the computation procedure are reported in Chuech et al.[15]

Ponstein[3] applied his linear stability analysis to the potential flow given by

$$U = U_0, \qquad V = \frac{A}{r}, \qquad W = 0 \qquad (0 \le r \le a) \tag{5}$$

where U_0 is the basic axial velocity in the jet, V is the basic tangential (swirl) velocity, A is a constant that represents the strength of the free vortex, r is the radial coordinate, and W is the basic radial velocity in the jet.

For axisymmetric disturbances, Ponstein[3] gives

$$\frac{\omega_i}{\sqrt{(\sigma/\rho a^3)}} = \left[(s + 1 - ka)\frac{I_1(ka)}{I_0(ka)} ka\right]^{1/2} \tag{6}$$

where ω_i is the growth rate of the instability waves, s is the swirl Weber number $s = \rho A^2/(\sigma a)$, and I_0 and I_1 are the modified Bessel functions of the first kind of zeroth and first order, respectively. To be able to compare our results with those of Ponstein,[3] we employ the same basic state, given by Eq. (5), in our analysis of a swirling liquid jet. By substituting the basic state in the Navier-Stokes equations applicable to an inviscid incompressible jet, it is easily shown that

$$P - \frac{1}{2}\rho A^2\left(\frac{1}{r^2} - \frac{1}{a^2}\right) = P_g + P_\sigma \tag{7}$$

where P, P_g, and P_σ are the basic state pressures due to liquid, gas, and surface tension, respectively. Note that in this case $P_\sigma = \sigma/a$. In the perturbed state, Eq. (7) becomes

$$P + p - \frac{1}{2}\rho A^2\left[\frac{1}{(r+\eta)^2} - \frac{1}{a^2}\right] = P_g + p_g + P_\sigma + p_\sigma \tag{8}$$

where η is the perturbation in the radial direction. By subtracting Eq. (7) from Eq. (8) after substituting $r = a$ in both equations, we obtain

$$p - \frac{1}{2}\rho A^2\left(\frac{1}{h^2} - \frac{1}{a^2}\right) = p_g + p_\sigma \tag{9}$$

where $h = a + \eta$.

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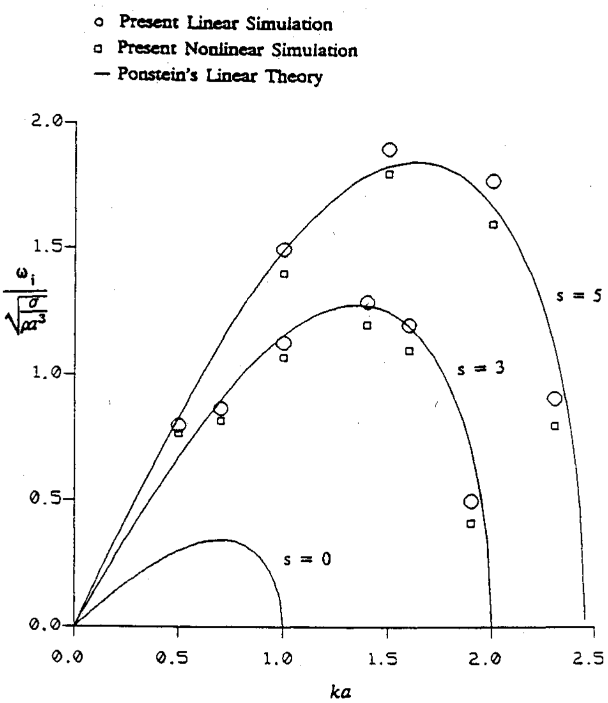

Fig. 1 Variation of wave growth rate with wave number.

Eq. (9) gives the modified form of Eq. (3) due to the presence of the swirl in the jet. After substituting from Eq. (9) into Eq. (2), the right-hand side of Eq. (2) becomes

$$-\frac{h^2}{\rho}\left(\frac{\partial p}{\partial z}\right)=-\frac{h^2}{\rho}\frac{\partial}{\partial z}(p_g+p_\sigma)+\frac{A^2}{h}\frac{\partial h}{\partial z} \tag{10}$$

So the result of introducing the swirl in the jet on the formulation of the problem is the addition of the term $(A^2/h)(\partial h/\partial z)$ to the right-hand side of Eq. (2).

Results and Discussion

Figure 1 shows a comparison of the predictions of the present theory and the results of Ponstein's[3] linear stability analysis when the dimensionless growth rate is plotted vs the dimensionless wave number. It is observed that excellent agreement is obtained between the linearized version of the present direct simulation and Ponstein's[3] theory. The linearization of the direct simulation is achieved by discarding the convective term in the momentum equation, Eq. (2), and simplifying the nonlinear form of the surface tension term given by Eq. (4) to

$$p_\sigma=-\frac{\sigma}{a^2}\left[(h-2a)+a^2\frac{\partial^2 h}{\partial z^2}\right] \tag{11}$$

Equation (11) is arrived at by assuming that

$$\eta=(h-a)\ll 1, \qquad \left(\frac{\partial h}{\partial z}\right)\ll 1 \tag{12}$$

The gas pressure p_g has been neglected in the computations.

It is seen from Fig. 1 that the higher the value of the swirl Weber number, the greater is the growth rate for a given wave number. A higher growth rate for the instability waves manifests itself in a greater instability of the jet. This is in agreement with the results of Kang and Lin.[4] At swirl Weber number $s=0$, Rayleigh's[17] results for the instability of an inviscid cylindrical jet in a vacuum that exhibits a maximum growth rate at $ka\simeq 0.7$ are reproduced. The excellent agreement between the predictions of the linearized version of the present direct simulation and those of Ponstein[3] establishes confidence in the accuracy of the current numerical computations.

The nonlinear direct simulation results are also plotted in Fig. 1. It is observed that the growth rates obtained through the nonlinear simulation are slightly lower than those obtained via the linear direct simulation. This trend has also been observed in the analyses associated with the Rayleigh jet.[7,15] Although the gas perturbation pressure p_g has been neglected in the previous results, it is interesting to envisage the effect of introducing a swirl in the gas. If a free vortex of velocity, $V_g=A_g/r$, is set up in the gas, Eq. (9) should be modified to read

$$p-\frac{1}{2}\rho A^2\left(\frac{1}{h^2}-\frac{1}{a^2}\right)=p_g-\frac{1}{2}\rho_g A_g^2\left(\frac{1}{h^2}-\frac{1}{a^2}\right)+p_\sigma \tag{13}$$

where ρ_g is the gas density.

Equation (13) suggests that the effect of the gas swirl will contradict that of the liquid jet swirl. Therefore, it may be concluded that swirling the gas would hinder the jet atomization. This is in complete agreement with the findings of Lian and Lin.[5]

It has also been found, via the present theory, that the liquid swirl causes the satellite drops to merge forward with main drops.

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